

Time: 6 minutes: Open book, open notes, calculator allowedSuppose the function f satisfies

$$\int_0^2 f(u) du = 10, \quad \int_0^4 f(u) du = 16.$$

Also, suppose that f is an **even** function.

Fill in each blank with a number (the value of the definite integral). Show your work, too.

$$\underline{10} \quad \int_2^4 f(x-2) dx = \int_0^2 f(u) du = 10$$

$$\left\{ \begin{array}{l} u = x - 2 \\ du = dx \\ x = 2 \implies u = 0 \\ x = 4 \implies u = 2 \end{array} \right\}$$

$$\underline{6} \quad \int_2^4 f(x) dx = \int_0^4 f(x) dx - \int_0^2 f(x) dx = 16 - 10 = 6$$

$$\underline{8} \quad \int_0^2 f(2x) dx = \frac{1}{2} \int_0^4 f(u) du = \left(\frac{1}{2}\right) \cdot 16 = 8$$

$$\left\{ \begin{array}{l} u = 2x \\ du = 2 dx \\ \frac{1}{2} du = dx \\ x = 0 \implies u = 0 \\ x = 2 \implies u = 4 \end{array} \right\}$$

$$\underline{8} \quad \int_0^2 f(x^2) \cdot x dx = \frac{1}{2} \int_0^4 f(u) du = \left(\frac{1}{2}\right) \cdot 16 = 8$$

$$\left\{ \begin{array}{l} u = x^2 \\ du = 2x dx \\ \frac{1}{2} du = x dx \\ x = 0 \implies u = 0 \\ x = 2 \implies u = 4 \end{array} \right\}$$

$$\underline{20} \quad \int_{-2}^2 f(x) dx = 2 \int_0^2 f(x) dx = 2 \cdot 10 = 20$$

We have used the fact that f is an even function.